# STATISTICS SELECTED FORMULAE

**PROBABILITY**

*conditional*: $P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$ *independent events*: $P(A \text{ and } B) = P(A) \cdot P(B)$ *dependent*: $P(A \text{ and } B) = P(A) \cdot P(B|A)$

*sample mean*: $\mu = \frac{\sum x}{N}$ *mean or expected value*: $\mu_X = \sum x \cdot P(x)$ *standard deviation*: $\sigma_X = \sqrt{\sum[(x - \mu_X)^2 \cdot P(x)]}$

$P(x) = \frac{f(x)}{\sum f(x)}$ *Hypergeometric probability*: $P(x) = \frac{C_k^x \cdot C_{N-k}^{n-x}}{C_N^n}$

**DISTRIBUTIONS**

*normal distribution*: $f(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$ *if* $\mu = 0$ *and* $\sigma = 1$, $f(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$ $\sigma \approx \frac{R_{ange}}{4}$

*standard score*: $z = \frac{X - \mu}{\sigma}$ *or* $z = \frac{X - \bar{X}}{s}$

***Binomial***: $\mu = n \cdot p$ $\sigma = \sqrt{n \cdot p \cdot q}$ $q = (1 - p)$ $P(x) = C_n^x \cdot p^x \cdot q^{n-x}$

***Binomial distribution***: $C_n^0 p^0 q^n + C_n^1 p^1 q^{n-1} + C_n^2 p^2 q^{n-2} + \cdots + C_n^{n-1} p^{n-1} q^1 + C_n^n p^n q^0$

***Poisson***: $\mu_X = \lambda \cdot t$ $\sigma_X = \sqrt{\lambda \cdot t}$ $P(x) = \frac{\mu^x}{x!} e^{-x}$ ***Geometric***: $\mu_X = \frac{1}{p}$ ***Uniform***: $\mu = \frac{a+b}{2}$ $\sigma = \sqrt{\frac{(b-a)^2}{12}}$

***Exponential***: $P(X < a) = 1 - e^{-\frac{a}{\mu}}$ ***Chi − square***: $\chi^2 = \frac{(n-1)s^2}{\sigma^2}$ *(for a population $\sigma$ and a sample $s$)*

**DISTRIBUTION OF SAMPLE MEANS**

*mean of sample means*: $\mu_{\bar{X}} = \mu$ *standard error*: $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$ *central limit*: $z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$ $n > 30$

*mean of sample proportions*: $\mu_{\bar{p}} = p$ $q = 1 - p$ *standard error*: $\sigma_{\bar{p}} = \sqrt{\frac{p \cdot q}{n}}$ *central limit*: $z = \frac{\bar{p} - \mu_{\bar{p}}}{\sigma_{\bar{p}}}$ $np > 5$ **and** $nq > 5$

*two samples expected value*: $E(\bar{x}_1 - \bar{x}_2) = \mu_{\bar{x}_1 - \bar{x}_2} = \mu_1 - \mu_2$ *standard error of* $\bar{x}_1 - \bar{x}_2$: $\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

**CONFIDENCE INTERVALS SAMPLE SIZES**

*sample size of means*: $n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2$ *margin of error E*: $\mu \pm E$ $E = z_{\alpha/2} \cdot \sqrt{\frac{p \cdot q}{n}}$

*sample size of proportions*: $n = \hat{p} \cdot \hat{q} \left(\frac{z_{\alpha/2}}{E}\right)^2$ **or** $n = 0.25 \left(\frac{z_{\alpha/2}}{E}\right)^2$ *(if $\hat{p}$ is unknown)* *margin of error E*: $p \pm E$

**REGRESSION**

*Least squares regression line*: $\hat{y} = ax + b$ $a = r \cdot \frac{\sigma_y}{\sigma_x}$ $b = \bar{y} - a\bar{x}$

**COMPOUND INTEREST**

$FV = PV\left(1 + \frac{r}{n}\right)^{n \cdot t}$ *Continuous*: $FV = PV \cdot e^{r \cdot t}$ *effective rate*: $r_e = e^r - 1$ (**r**: *annual rate* **n**: *payments per year* **t**: *years*)

**ANNUITY**

$i = \frac{r}{n}$; $PV = 0$: $FV = PMT \cdot \frac{(1+i)^{n \cdot t} - 1}{i}$ ; $FV = 0$: $PMT = PV \cdot \frac{i(1+i)^{n \cdot t}}{(1+i)^{n \cdot t} - 1}$

**NOTES**

**Rounding practice**. Round statistics, the mean, standard deviation and variance to one more decimal place than the values of the original data. (Sullivan pp 326, 440)
**On TI-84 TVM: END:** payment at the end of the period**,** for regular annuities (FV=0) or amortization (PV=0).
**BEGIN**: payment at the beginning of the period, for annuities due (PV=0).

# POKER SELECTED PROBABILITIES

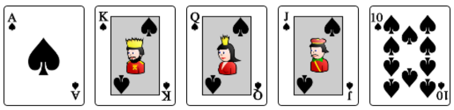
$$P(AH) = \frac{C_1^1}{C_{52}^1} = \frac{1}{52} = 0.0192 \quad \textit{an ace of heatrs}$$

$$P(10) = \frac{C_4^1}{C_{52}^1} = \frac{4}{52} = 0.0769 \quad a\ 10$$

$$P(3K) = \frac{C_4^3}{C_{52}^3} = \frac{4}{22100} = 0.000181 \quad 3\ kings$$

$$P(BQ\ and\ H) = \frac{C_2^1 \cdot C_{13}^1}{C_{52}^2} = \frac{26}{2652} = 0.00980 \quad \textit{a black queen and a heart}$$

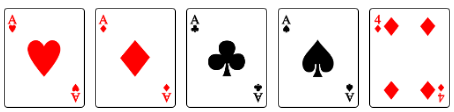
**ROYAL FLUSH**



$$P(A,K,Q,J,10) = \frac{C_4^1}{C_{52}^5} = \frac{4}{2598960} = 0.00000154 \quad \textit{royal flush}, (\textit{same suit})$$

There are only four possible royal flushes. A, K, Q, J and 10 of the same suit.

**FOUR OF A KIND**

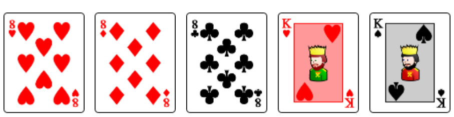


$$P(4\ of\ a\ kind) = \frac{C_{13}^1 \cdot C_{48}^1}{C_{52}^5} = \frac{13 \cdot 48}{2598960} = \frac{624}{2598960} = 0.000240$$

Four of a kind is four cards with same rank and some other card. Examples: K, K, K, K,5. 3,3,3,3,7.

$C_{13}^1$ selects 1 of the 13 ranks (13 choices). $C_{48}^1$ selects any of the remaining cards (48 choices).

**FULL HOUSE**



$$P(full\ house) = \frac{C_{13}^1 \cdot C_4^3 \cdot C_{12}^1 \cdot C_4^2}{C_{52}^5} = \frac{13 \cdot 4 \cdot 12 \cdot 6}{2598960} = \frac{3744}{2598960} = 0.00144$$

Full house is three the same rank and a pair. The pair is two of the same rank. Examples: Q, Q, Q, 3,3. 10, 10, 10, 5,5.

$C_{13}^1$ selects 1 of the 13 ranks (13 choices). $C_4^3$ selects 3 of the four possibilities within the rank (4 choices) . $C_{12}^1$ selects one of the remaining 12 ranks (12 choices) and $C_4^2$ selects 2 of the 4 possibilities (6 choices).

**BIRTHDAY PROBLEM**

*Probability that in a group of n people, at least 2 have the same birthday*:

$$P(n) = 1 - \frac{365!}{365^n \cdot (365-n)!} \quad \textit{example}: \quad P(5) = 1 - \frac{365 \cdot 364 \cdot 363 \cdot 362 \cdot 361}{365^5} = 0.0271 \quad P(25) = 0.569$$

# COUNTING

**FUNDAMENTAL COUNTING RULE**

$$Count = k_1 \cdot k_2 \cdot k_3 \cdot k_4 \cdots k_n$$

$$Example:\ \ choices = 4\ of\ A,\ \ 2\ of\ B,\ \ 2\ of\ C\ \ and\ \ 3\ of\ D \qquad Count = 4 \cdot 2 \cdot 2 \cdot 3 = 48$$

**SELECTION**

DISTINCT – WITH REPETITION – ORDERED

$$Number\ of\ choices\ =\ n;\ \ Number\ of\ selections\ =\ r \qquad Count\ = n^r\ \ or\ \ Count\ = n^n\ \ (if\ \ r = n)$$

$$Example:\ \ A\ B\ C\ D \qquad n = 4,\ \ r = 2,\quad n^r = 4^2 = 16$$

| | | | |
|---|---|---|---|
| AA | BA | CA | DA |
| AB | BB | CB | DB |
| AC | BC | CC | DC |
| AD | BD | CD | DD |

**PERMUTATION**

DISTINCT – WITHOUT REPETITION – ORDERED

$$_nP_r\ = \frac{n!}{(n-r)!} \qquad or \qquad _nP_n\ = n!\ \ \ (if\ \ r = n)$$

$$Example:\ \ A\ B\ C\ D \qquad n = 4,\ \ r = 2, \qquad _4P_2 = \frac{4!}{(4-2)!} = 12$$

| | | | |
|---|---|---|---|
| AB | BA | CA | DA |
| AC | BC | CB | DB |
| AD | BD | CD | DC |

**PERMUTATION k**

NON DISTINCT – USE ALL

$$_nP_k\ = \frac{n!}{n_1! \cdot n_2! \ldots n_k!}$$

$$Example:\ \ A\ A\ A\ B\ C\ C \qquad n = 6,\ n_1 = 3,\ \ n_2 = 1,\ \ n_3 = 2 \qquad \frac{n!}{n_1! \cdot n_2! \ldots n_k!} = \frac{6!}{3! \cdot 1! \cdot 2!} = 60$$

| | | | |
|---|---|---|---|
| AAABCC | BAACAC | CBAACA | … |
| AABACC | BACAAC | CBACAA | … |
| ABAACC | BCAAAC | CBCAAA | … |
| BAAACC | CBAAAC | CCBAAA | … |

**COMBINATION**

DISTINCT – WITHOUT REPETITION –NOT ORDERED

$$_nC_r\ = \frac{n!}{(n-r)! \cdot r!}$$

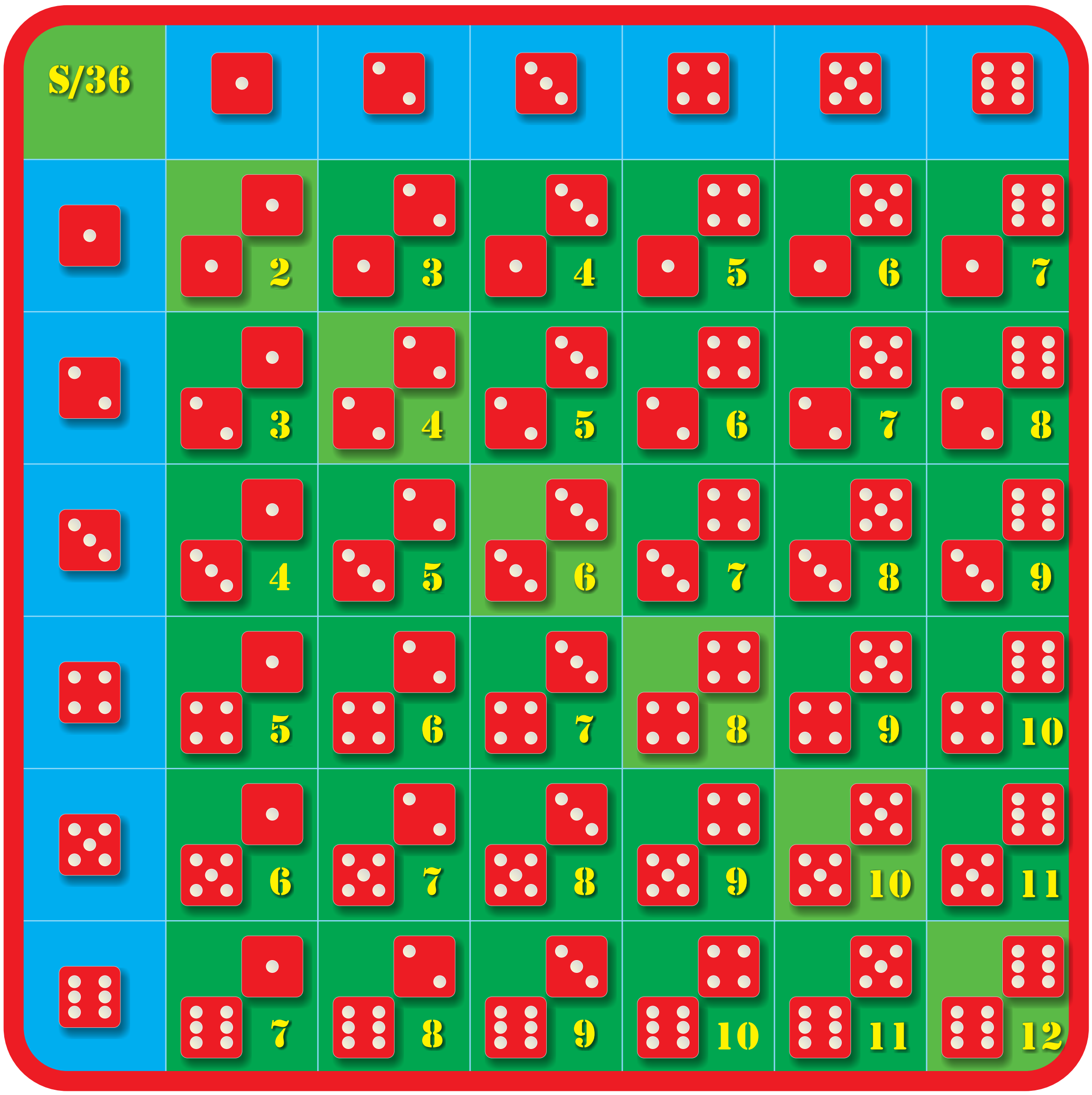
$$Example:\ \ A\ B\ C\ D\ E\ F \qquad n = 6,\ \ r = 2, \qquad _6C_2 = \frac{6!}{(6-2)! \cdot 2!} = 15$$

| | | | | |
|---|---|---|---|---|
| AB | AC | AD | AE | AF |
| BC | BD | BE | BF | CD |
| CE | CF | DE | DF | EF |

# PLAYING CARDS

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A♥ | 2♥ | 3♥ | 4♥ | 5♥ | 6♥ | 7♥ | 8♥ | 9♥ | 10♥ | J♥ | Q♥ | K♥ |
| A♦ | 2♦ | 3♦ | 4♦ | 5♦ | 6♦ | 7♦ | 8♦ | 9♦ | 10♦ | J♦ | Q♦ | K♦ |
| A♠ | 2♠ | 3♠ | 4♠ | 5♠ | 6♠ | 7♠ | 8♠ | 9♠ | 10♠ | J♠ | Q♠ | K♠ |
| A♣ | 2♣ | 3♣ | 4♣ | 5♣ | 6♣ | 7♣ | 8♣ | 9♣ | 10♣ | J♣ | Q♣ | K♣ |

# TWO DICE THROW

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