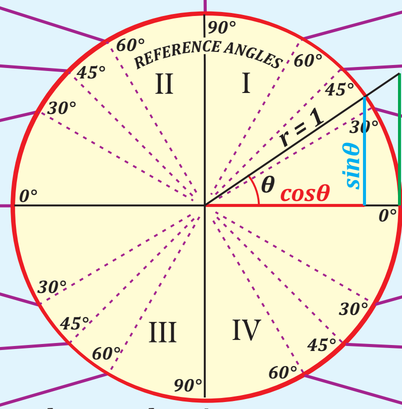
TRIGONOMETRY
AND MATH REFERENCE

CHART MTR103 bernie649@fastmail.fm MATH INSTRUCTION - GRAPHIC DESIGN

COMMON ANGLES AND FUNCTIONS

tan	cos	sin	rad	deg
$-\sqrt{3}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{2}{3}\pi$	120°
-1	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{3}{4}\pi$	135°
$-\frac{\sqrt{3}}{3}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{5}{6}\pi$	150°
0	-1	0	π	180°
$\frac{\sqrt{3}}{3}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{7}{6}\pi$	210°
1	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	$\frac{5}{4}\pi$	225°
$\sqrt{3}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{4}{3}\pi$	240°

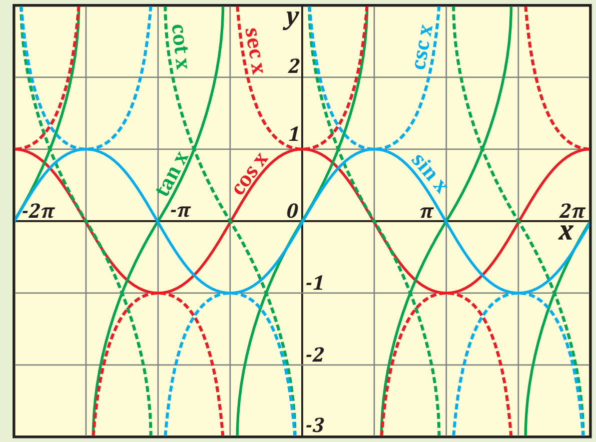
deg	rad	sin	cos	tan
90°	$\frac{1}{2}\pi$	1	0	∅



deg	rad	sin	cos	tan
270°	$\frac{3}{2}\pi$	-1	0	∅

deg	rad	sin	cos	tan
60°	$\frac{1}{3}\pi$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
45°	$\frac{1}{4}\pi$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
30°	$\frac{1}{6}\pi$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
0° 360°	0 2π	0	1	0
330°	$\frac{11}{6}\pi$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$
315°	$\frac{7}{4}\pi$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1
300°	$\frac{5}{3}\pi$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$

THE SIX FUNCTIONS



TRIGONOMETRIC IDENTITIES

FUNDAMENTAL IDENTITIES

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta \quad \cos^2\theta = 1 - \sin^2\theta$

$\sec^2\theta = 1 + \tan^2\theta \quad \csc^2\theta = 1 + \cot^2\theta$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} \quad \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$\csc\theta = \dfrac{1}{\sin\theta} \quad \sec\theta = \dfrac{1}{\cos\theta}$

PRODUCT OR SUM

$2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$

$2\cos\alpha\sin\beta = \sin(\alpha+\beta) - \sin(\alpha-\beta)$

$2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$

$2\sin\alpha\sin\beta = \cos(\alpha-\beta) - \cos(\alpha+\beta)$

$\sin\alpha + \sin\beta = 2\sin\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$

$\sin\alpha - \sin\beta = 2\cos\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}$

$\cos\alpha + \cos\beta = 2\cos\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$

$\cos\alpha - \cos\beta = -2\sin\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}$

SUM OR DIFFERENCE OF TWO ANGLES

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

REFLECTIVE

$\sin(\pi-\theta) = \sin\theta \quad \cos(\pi-\theta) = -\cos\theta$

$\tan(\pi-\theta) = -\tan\theta \quad \cos\left(\dfrac{\pi}{2}+\theta\right) = -\sin\theta$

$\sin\left(\dfrac{\pi}{2}-\theta\right) = \cos\theta \quad \cos\left(\dfrac{\pi}{2}-\theta\right) = \sin\theta$

$\tan\left(\dfrac{\pi}{2}-\theta\right) = \cot\theta \quad \sin\left(\dfrac{\pi}{2}+\theta\right) = \cos\theta$

HALF ANGLE

$\tan\dfrac{\theta}{2} = \dfrac{\sin\theta}{1+\cos\theta} = \dfrac{1-\cos\theta}{\sin\theta} = \pm\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}$

$\sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{2}} \quad \cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$

POWER REDUCING

$\sin^2\alpha = \dfrac{1-\cos 2\alpha}{2} \quad \cos^2\alpha = \dfrac{1+\cos 2\alpha}{2}$

$\tan^2\alpha = \dfrac{1-\cos 2\alpha}{1+\cos 2\alpha} \quad \sin^2\alpha\cos^2\alpha = \dfrac{1-\cos 4\alpha}{8}$

$\sin^3\theta = \dfrac{3\sin\theta - \sin 3\theta}{4}$

$\cos^3\theta = \dfrac{3\cos\theta + \cos 3\theta}{4}$

$\sin^3\theta\cos^3\theta = \dfrac{3\sin 2\theta - \sin 6\theta}{32}$

DOUBLE OR TRIPLE ANGLES

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos 2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$

$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta = 3\sin\theta - 4\sin^3\theta$

$\cos 3\theta = \cos^3\theta - \sin^2\theta\cos\theta = 4\cos^3\theta - 3\cos\theta$

$\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta} \quad \cot 3\theta = \dfrac{3\cot\theta - \cot^3\theta}{1 - 3\cot^2\theta}$

NEGATIVE

$\sin(-\theta) = -\sin\theta \quad \cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta \quad \cot(-\theta) = -\cot\theta$

$\csc(-\theta) = -\csc\theta \quad \sec(-\theta) = \sec\theta$

INVERSE FUNCTIONS

$\sin(\sin^{-1}x) = x \quad \sin^{-1}(\sin\theta) = \theta$

$\cos(\cos^{-1}x) = x \quad \cos^{-1}(\cos\theta) = \theta$

$\tan(\tan^{-1}x) = x \quad \tan^{-1}(\tan\theta) = \theta$

$\sin^{-1}(\cos\theta) = \cos^{-1}(\sin\theta)$

$\tan^{-1}(\cot\theta) = \cot^{-1}(\tan\theta)$

$\sin(\cos^{-1}x) = \cos(\sin^{-1}x) = \sqrt{1-x^2}$

$\sin(\tan^{-1}x) = \cos(\cot^{-1}x) = \dfrac{x}{\sqrt{1+x^2}}$

$\tan(\sin^{-1}x) = \cot(\cos^{-1}x) = \dfrac{x}{\sqrt{1-x^2}}$

$\cos(\tan^{-1}x) = \sin(\cot^{-1}x) = \dfrac{1}{\sqrt{1+x^2}}$

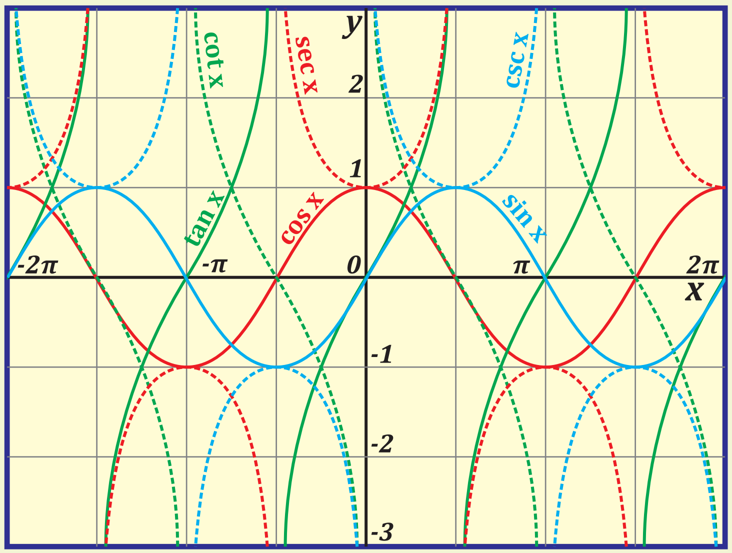
$\tan(\cos^{-1}x) = \cot(\sin^{-1}x) = \dfrac{\sqrt{1-x^2}}{x}$

$\tan(\cot^{-1}x) = \cot(\tan^{-1}x) = \dfrac{1}{x}$

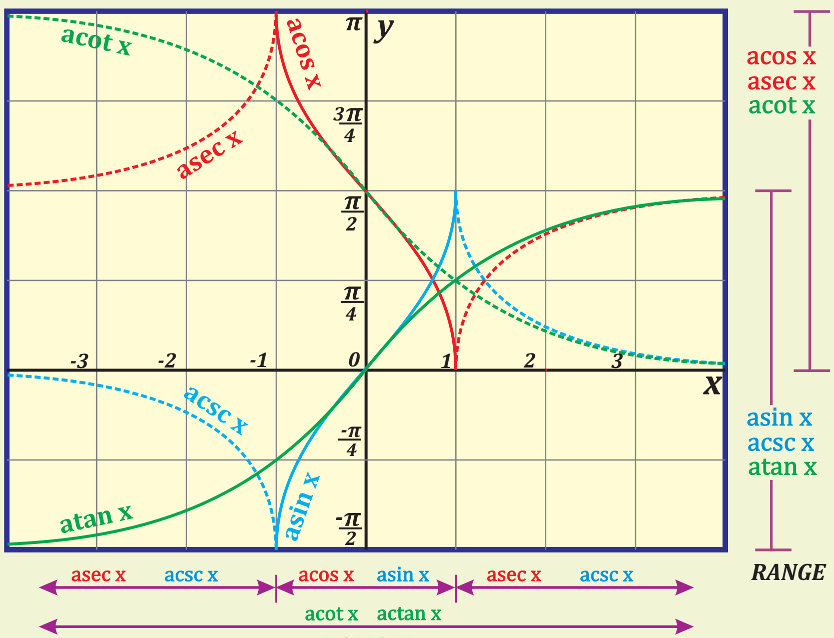
TRIGONOMETRY REFERENCE II

www.science31415.com bernie649@fastmail.fm

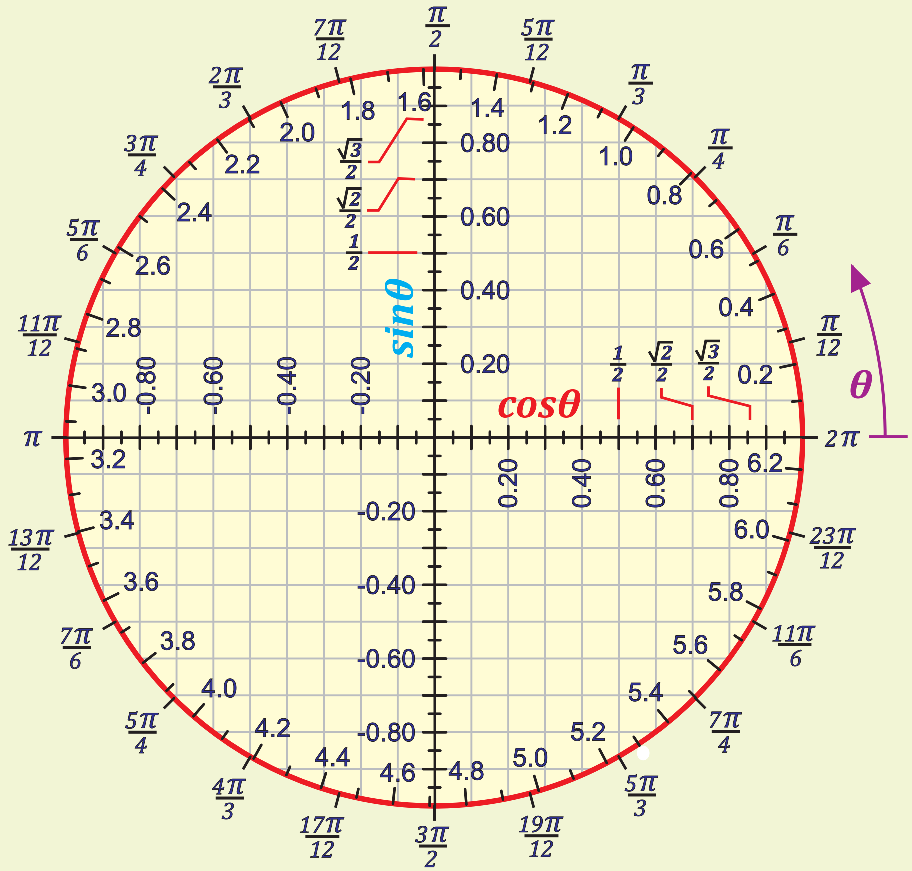
FUNCTIONS



INVERSE FUNCTIONS



THE UNIT CIRCLE



$$A\cos\vartheta + B\sin\vartheta = C\sin(\vartheta + \varphi)$$

$$C = \sqrt{A^2 + B^2};\quad \varphi = arctan\left(\frac{A}{B}\right)$$

COSINE SINE VECTOR ADDITION