EQUATIONS

QUADRATIC EQUATION

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

SPECIAL PRODUCTS

$$a^2 - b^2 = (a + b)(a - b)$$

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$a^2 + b^2 = (a + bi)(a - bi)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

COMPLEX IDENTITIES

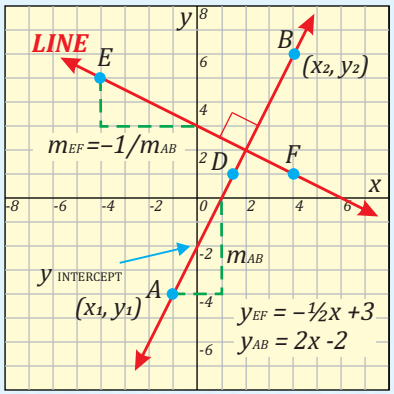
$$i = \sqrt{-1} \quad i^2 = -1 \quad i^3 = -i \quad i^4 = 1$$

$$i^5 = i \quad \sqrt{i} = \frac{\sqrt{2}}{2}(1 + i) \quad e^{i\pi} = -1$$

$$(a + bi)^2 = a^2 - b^2 + 2abi$$

$$(a - bi)^2 = a^2 - b^2 - 2abi$$

EQUATIONS IN GRAPHIC FORM

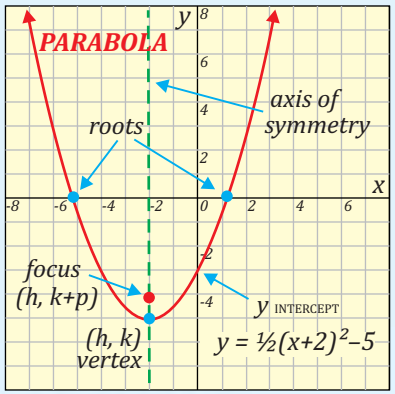


$$y = mx + b \quad or: \; y - y_1 = m(x - x_1)$$

$$D: \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right) \quad m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$AB = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

m: slope, b: y intercept, D: mid point,
AB: distance between two points.

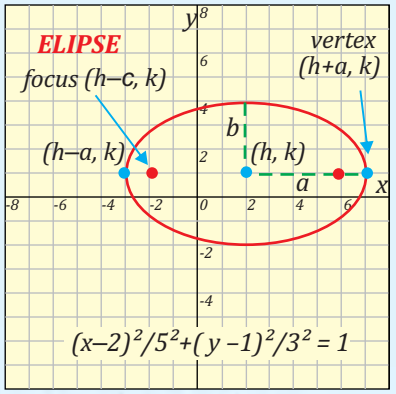


$$y = a(x - h)^2 + k$$

$$(x - h)^2 = 4p(y - k)$$

$$y = ax^2 + bx + c \quad h = -\frac{b}{2a}$$

Three equivalent parabola equations,
p: focus distance from vertex, x=h: axis.

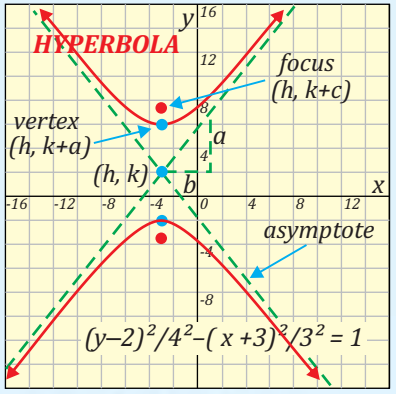


$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

$$c^2 = a^2 - b^2 \quad e = \frac{c}{a}$$

$$(x - h)^2 + (y - k)^2 = r^2 \text{ (circle)}$$

2a: major axis, 2b: minor axis,
c: focus distance, e: eccentricity.



$$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$$

$$c^2 = a^2 + b^2 \quad e = \frac{c}{a}$$

$$y - k = \pm\frac{a}{b}(x - h) \text{ (asymptotes)}$$

a: vertex to center distance, c: focus,
a/b: asymptote slope, e: eccentricity.

NUMBER SYSTEM

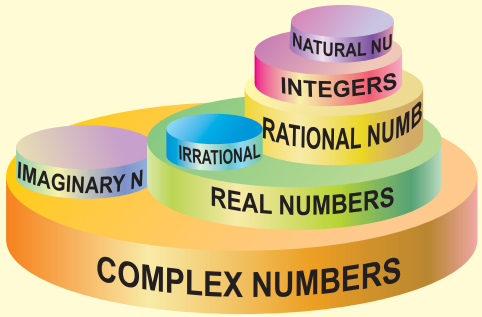
NATURAL NUMBERS $\mathbb{N}$
Also called counting numbers or whole numbers.
$1, 2, 3, 4, 5 \ldots$

INTEGERS $\mathbb{Z}$
Include the natural numbers plus zero and negative whole numbers.
$\ldots -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5 \ldots$

RATIONAL NUMBERS $\mathbb{Q}$
Expressed as a ratio of two integers a/b. As decimals they terminate or continue in a repeating pattern.
Examples:
$\frac{3}{4} = 0.75 \quad \frac{27}{110} = 0.245\,45\,45\,45 \ldots = 0.2\overline{45}$

IRRATIONAL NUMBERS
Numbers that cannot be expressed as a ratio of two integers a/b. As decimals they are non-terminating, continuing indefinitely without a repeating pattern.
Examples:
$\sqrt{2}, \quad \pi = 3.14159265358979323846264 \ldots$

REAL NUMBERS $\mathbb{R}$
Include the natural numbers, the integers, the rational and the irrational numbers.

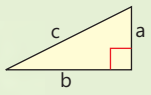


IMAGINARY NUMBERS
The square root of a negative number. Expressed as:
$bi \;\; where \; i = \sqrt{-1} \;\; and \;\; i^2 = -1$

COMPLEX NUMBERS $\mathbb{C}$
Include real and imaginary numbers. Expressed as:
$a + bi$
If b = 0 the number is real, if a = 0 the number is imaginary if both a and b are not zero the number is complex.

TRIANGLE EQUATIONS

PYTHAGOREAN THEOREM



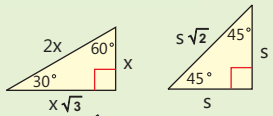
$$c^2 = a^2 + b^2$$

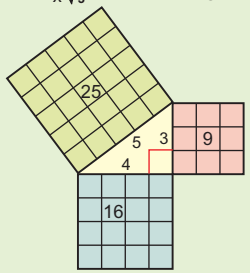
$$c = \sqrt{a^2 + b^2}$$

$$a = \sqrt{c^2 - b^2}$$

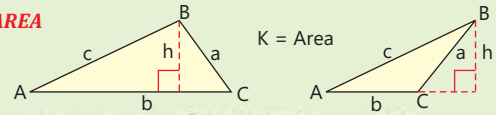
$$b = \sqrt{c^2 - a^2}$$

PYTHAGOREAN TRIPLETS		
a	b	c
3	4	5
5	12	13
20	21	29
28	45	53





AREA

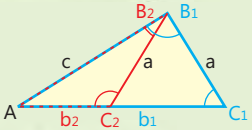


$$K = \frac{1}{2}ab\,sinC = \frac{a^2 sinB\,sinC}{2sinA} \qquad K = \frac{bh}{2}$$

$$K = \sqrt{s(s-a)(s-b)(s-c)} \qquad s = \frac{a + b + c}{2}$$

LAW OF SINES

$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$$



ASS, two resulting triangles:

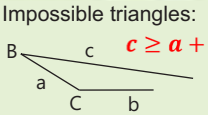
$$sin^{-1}\left(\frac{c\,sinA}{a}\right) = C_1 \; or \; C_2 \qquad C_2 = \pi - C_1$$

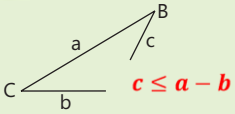
LAW OF COSINES AND TRIANGLE INEQUALITIES

$$c^2 = a^2 + b^2 - 2ab\,cosC \qquad |a - b| < c < a + b$$

$$|cos\,C| = \left|\frac{a^2 + b^2 - c^2}{2ab}\right| < 1$$

Impossible triangles:





EXPONENTS AND LOGARITHMS

Exponential Properties

$a^x \cdot a^y = a^{x+y}$

$\frac{a^m}{a^n} = a^{m-n}$

$(a^m)^n = a^{mn}$

$(a^m \cdot b^m) = (ab)^m$

$\sqrt[n]{a} = a^{\frac{1}{n}}$

$\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m = a^{\frac{m}{n}}$

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

$(a^m \cdot b^n \cdot c)^k = a^{mk} \cdot b^{nk} \cdot c^k$

Logarithmic Properties

$a^y = x \ \ \therefore \ \ log_a x = y$

$log_a x = y \ \ \therefore \ \ a^y = x$

$log_{10} x = log\, x$

$log_e x = ln\, x$

$a^{log_a x} = x$

$a^{\frac{1}{lna}} = e$

$a^{lne} = a$

$a^b = e^{b \cdot lna}$

$a^{lnb} = b^{lna}$

$e^{ln\, x} = x$

$log_a a = 1$

$log_a 1 = 0$

$log_a a^x = x$

$ln\, e^x = x$

$ln\, xy = ln\, x + ln\, y$

$ln\frac{x}{y} = ln\, x - ln\, y$

$ln\, x^y = y\, ln\, x$

$log_a x = \frac{log_{10} x}{log_{10} a} = \frac{ln\, x}{ln\, a}$

$log_a b = c$

$log_a \frac{1}{b} = -log_a b = -c$

Exponents

$a = a^1 = a$

$a^0 = 1$

$a \cdot a = a^2$

$a \cdot a \cdot a = a^3$

$a \cdot a \cdot a \cdot a \cdot a \cdot a = a^6$

$\frac{1}{a} = a^{-1}$

$\frac{1}{a^3} = a^{-3}$

$\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$

$\frac{b^{-2}}{a^{-3}} = \frac{a^3}{b^2}$

$\frac{1}{(a+2b)} = (a+2b)^{-1}$

$(a-b^2)^{-3} = \frac{1}{(a-b^2)^3}$

Radicals

$\left(\sqrt{a}\right)^2 = \sqrt{a^2} = |a|$

$\sqrt{a} \cdot \sqrt{a} = |a|$

$\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$

$\sqrt{a} = \sqrt[2]{a} = a^{\frac{1}{2}}$

$\sqrt[3]{a} = a^{\frac{1}{3}}$

$\sqrt[5]{a^2} = a^{\frac{2}{5}}$

$a^2 \cdot \sqrt{a^5} = \sqrt{a^4 \cdot a^5} = \sqrt{a^9}$

$\sqrt{a^7} = \sqrt{a^6 \cdot a} = a^3 \cdot \sqrt{a}$

$\frac{1}{\sqrt{a^5}} = a^{-\frac{5}{2}}$

Addition of similar terms:

(add the coefficients)

$2a + 3a^3 - a + 4a^3 = a + 7a^3$

$3ab^2 + 5b + 4ab^2 = 7ab^2 + 5b$

$5ab - 3c - 2ab - c = 3ab - 4c$

Multiplication: *(add the exponents)*

$a \cdot a^2 = a^{1+2} = a^3$

$a^4 \cdot a^2 = a^{4+2} = a^6$

$a^3 \cdot b^5 \cdot b^4 \cdot a = a^4 b^9$

$b^2(ab^3 + a^2 b) = ab^5 + a^2 b^3$

$a^2 b^4 (a^2 b^2 + 5a) = a^4 b^6 + 5a^3 b^4$

$a^{\frac{1}{2}} \cdot a \cdot a^2 = a^{\frac{1}{2}+1+2} = a^{\frac{7}{2}}$

$a \cdot a^{-\frac{3}{2}} \cdot a^5 = a^{1-\frac{3}{2}+5} = a^{\frac{9}{2}}$

$\frac{a}{b^2} = ab^{-2}; \ \ \frac{a^5}{a^3} = a^{5-3} = a^2$

Power of Exponents: *(multiply the exponents)*

$(a^3)^2 = a^{3 \cdot 2} = a^6$

$(a^2 b^3)^4 = a^8 b^{12}$

$(a^4 b^{-5})^{-2} = a^{-8} b^{10} = \frac{b^{10}}{a^8}$

$\left(\sqrt{a}\right)^2 = \left(a^{\frac{1}{2}}\right)^2 = a^{\frac{2}{2}} = a$

$\left(\sqrt{a}\right)^3 = \sqrt{a^3} = a^{\frac{3}{2}}$

$\left(\sqrt[3]{a^2 \cdot b^5}\right)^2 = \sqrt[3]{a^4 \cdot b^{10}} = a^{\frac{4}{3}} \cdot b^{\frac{10}{3}}$

Special Products

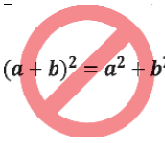
$a^2 - b^2 = (a+b)(a-b)$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$



Quadratic Equation

$ax^2 + bx + c = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminant: $b^2 - 4ac$

Axis of symmetry: $x = -\frac{b}{2a}$

MULTIPLICATION TABLE

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50
3	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75
4	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96	100
5	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125
6	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120	126	132	138	144	150
7	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140	147	154	161	168	175
8	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160	168	176	184	192	200
9	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180	189	198	207	216	225
10	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250
11	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220	231	242	253	264	275
12	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288	300
13	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260	273	286	299	312	325
14	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280	294	308	322	336	350
15	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360	375
16	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320	336	352	368	384	400
17	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340	357	374	391	408	425
18	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360	378	396	414	432	450
19	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380	399	418	437	456	475
20	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
21	21	42	63	84	105	126	147	168	189	210	231	252	273	294	315	336	357	378	399	420	441	462	483	504	525
22	22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352	374	396	418	440	462	484	506	528	550
23	23	46	69	92	115	138	161	184	207	230	253	276	299	322	345	368	391	414	437	460	483	506	529	552	575
24	24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	384	408	432	456	480	504	528	552	576	600
25	25	50	75	100	125	150	175	200	225	250	275	300	325	350	375	400	425	450	475	500	525	550	575	600	625

DIVISIBILITY

DIVISIBLE BY 2:
Even numbers.

DIVISIBLE BY 3:
The digits sum is a multiple of 3. 1275: 1+2+7+5 = 15.

DUVISIBLE BY 5:
Numbers ending in 0 or 5.

DIVISIBLE BY 7:
Subtract 2 times the last digit from the rest. 483: 48 − (3 × 2) = 42 = 7 × 6.

DIVISIBLE BY 11:
Subtract the last digit from the rest. 627: 62 − 7 = 55.

PARITY

- even × even = even;
- even × odd = even;
- odd × odd = odd.
- even ± even = even;
- even ± odd = odd;
- odd ± odd = even;

- **Even** = $\{2k; \forall k \in \mathbb{Z}\}$
- **Odd** = $\{2k+1; \forall k \in \mathbb{Z}\}$

If a quotient is an integer, it will be even if and only if the dividend has more factors of two than the divisor.

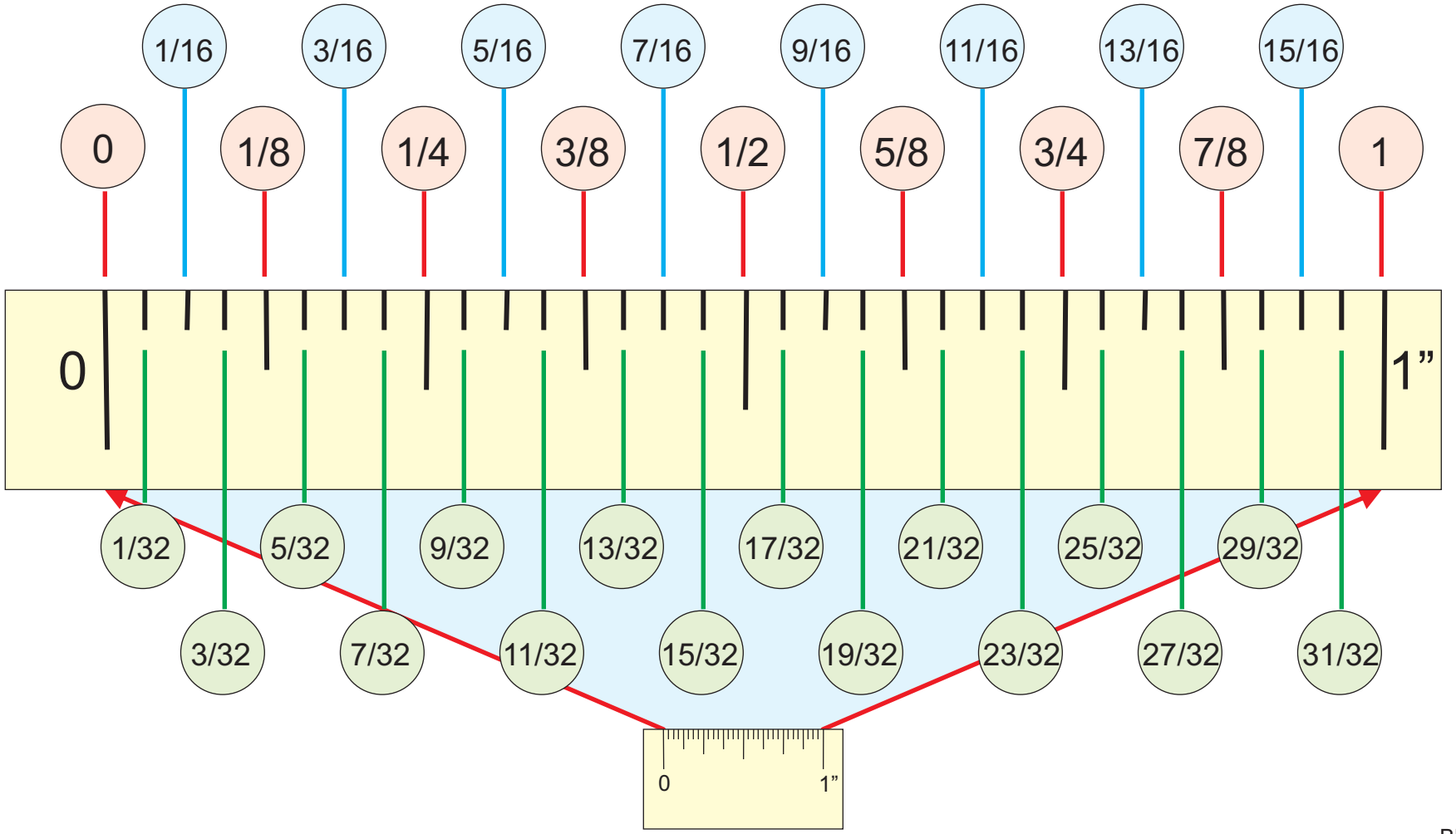
BM

PRIME NUMBERS

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1-20	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71
21-40	73	79	83	89	97	101	103	107	109	113	127	131	137	139	149	151	157	163	167	173
41-60	179	181	191	193	197	199	211	223	227	229	233	239	241	251	257	263	269	271	277	281
61-80	283	293	307	311	313	317	331	337	347	349	353	359	367	373	379	383	389	397	401	409
81-100	419	421	431	433	439	443	449	457	461	463	467	479	487	491	499	503	509	521	523	541
101-120	547	557	563	569	571	577	587	593	599	601	607	613	617	619	631	641	643	647	653	659
121-140	661	673	677	683	691	701	709	719	727	733	739	743	751	757	761	769	773	787	797	809
141-160	811	821	823	827	829	839	853	857	859	863	877	881	883	887	907	911	919	929	937	941
161-180	947	953	967	971	977	983	991	997	1009	1013	1019	1021	1031	1033	1039	1049	1051	1061	1063	1069
181-200	1087	1091	1093	1097	1103	1109	1117	1123	1129	1151	1153	1163	1171	1181	1187	1193	1201	1213	1217	1223
201-220	1229	1231	1237	1249	1259	1277	1279	1283	1289	1291	1297	1301	1303	1307	1319	1321	1327	1361	1367	1373
221-240	1381	1399	1409	1423	1427	1429	1433	1439	1447	1451	1453	1459	1471	1481	1483	1487	1489	1493	1499	1511
241-260	1523	1531	1543	1549	1553	1559	1567	1571	1579	1583	1597	1601	1607	1609	1613	1619	1621	1627	1637	1657
261-280	1663	1667	1669	1693	1697	1699	1709	1721	1723	1733	1741	1747	1753	1759	1777	1783	1787	1789	1801	1811
281-300	1823	1831	1847	1861	1867	1871	1873	1877	1879	1889	1901	1907	1913	1931	1933	1949	1951	1973	1979	1987
301-320	1993	1997	1999	2003	2011	2017	2027	2029	2039	2053	2063	2069	2081	2083	2087	2089	2099	2111	2113	2129
321-340	2131	2137	2141	2143	2153	2161	2179	2203	2207	2213	2221	2237	2239	2243	2251	2267	2269	2273	2281	2287
341-360	2293	2297	2309	2311	2333	2339	2341	2347	2351	2357	2371	2377	2381	2383	2389	2393	2399	2411	2417	2423
361-380	2437	2441	2447	2459	2467	2473	2477	2503	2521	2531	2539	2543	2549	2551	2557	2579	2591	2593	2609	2617
381-400	2621	2633	2647	2657	2659	2663	2671	2677	2683	2687	2689	2693	2699	2707	2711	2713	2719	2729	2731	2741
401-420	2749	2753	2767	2777	2789	2791	2797	2801	2803	2819	2833	2837	2843	2851	2857	2861	2879	2887	2897	2903
421-440	2909	2917	2927	2939	2953	2957	2963	2969	2971	2999	3001	3011	3019	3023	3037	3041	3049	3061	3067	3079
441-460	3083	3089	3109	3119	3121	3137	3163	3167	3169	3181	3187	3191	3203	3209	3217	3221	3229	3251	3253	3257
461-480	3259	3271	3299	3301	3307	3313	3319	3323	3329	3331	3343	3347	3359	3361	3371	3373	3389	3391	3407	3413
481-500	3433	3449	3457	3461	3463	3467	3469	3491	3499	3511	3517	3527	3529	3533	3539	3541	3547	3557	3559	3571

INCH FRACTIONS

1/32	2/32	3/32	4/32	5/32	6/32	7/32	8/32	9/32	10/32	11/32	12/32	13/32	14/32	15/32	16/32	17/32	18/32	19/32	20/32	21/32	22/32	23/32	24/32	25/32	26/32	27/32	28/32	29/32	30/32	31/32	32/32
1/32	1/16	3/32	1/8	5/32	3/16	7/32	1/4	9/32	5/16	11/32	3/8	13/32	7/16	15/32	1/2	17/32	9/16	19/32	5/8	21/32	11/16	23/32	3/4	25/32	13/16	27/32	7/8	29/32	15/16	31/32	1
0.03125	0.06250	0.09375	0.12500	0.15625	0.18750	0.21875	0.25000	0.28125	0.31250	0.34375	0.37500	0.40625	0.43750	0.46875	0.50000	0.53125	0.56250	0.59375	0.62500	0.65625	0.68750	0.71875	0.75000	0.78125	0.81250	0.84375	0.87500	0.90625	0.93750	0.96875	1.00000



BM

NAME	POWERS OF TEN	METRIC UNIT	SYM.	OBJECTS
	10^{29} 100,000,000,000,000,000,000,000,000,000			the Universe
	10^{28} 10,000,000,000,000,000,000,000,000,000			
octillion	10^{27} 1,000,000,000,000,000,000,000,000,000			
	10^{26} 100,000,000,000,000,000,000,000,000			
	10^{25} 10,000,000,000,000,000,000,000,000			
septillion	10^{24} 1,000,000,000,000,000,000,000,000	yottameter	Ym	galax.cluster
	10^{23} 100,000,000,000,000,000,000,000			
	10^{22} 10,000,000,000,000,000,000,000			
sextillion	10^{21} 1,000,000,000,000,000,000,000	zetameter	Zm	galaxy
	10^{20} 100,000,000,000,000,000,000			
	10^{19} 10,000,000,000,000,000,000			
quintillion	10^{18} 1,000,000,000,000,000,000	exameter	Em	
	10^{17} 100,000,000,000,000,000			near star
	10^{16} 10,000,000,000,000,000			light year
quadrillion	10^{15} 1,000,000,000,000,000	petameter	Pm	
	10^{14} 100,000,000,000,000			
	10^{13} 10,000,000,000,000			solar system
trillion	10^{12} 1,000,000,000,000	terameter	Tm	Juptr orbit
	10^{11} 100,000,000,000			Merc orbit
	10^{10} 10,000,000,000			
billion	10^{9} 1,000,000,000	gigameter	Gm	Sun
	10^{8} 100,000,000			Jupiter
	10^{7} 10,000,000			Earth
million	10^{6} 1,000,000	megameter	Mm	ocean
	10^{5} 100,000			country
	10^{4} 10,000			city
thousand	10^{3} 1,000	kilometer	km	airport
hundred	10^{2} 100	hectometer	hm	building
ten	10^{1} 10	decameter	dam	house
unit	10^{0} 1	meter	m	human
tenth	10^{-1} 0.1	decimeter	dm	mouse
hundredth	10^{-2} 0.01	centimeter	cm	bee
thousandth	10^{-3} 0.001	millimeter	mm	flea
	10^{-4} 0.000 1			hair
	10^{-5} 0.000 01			cell
millionth	10^{-6} 0.000 001	micrometer	μ	bacteria
	10^{-7} 0.000 000 1			virus
	10^{-8} 0.000 000 01			DNA
billionth	10^{-9} 0.000 000 001	nanometer	nm	molecule
	10^{-10} 0.000 000 000 1			Gold atom
	10^{-11} 0.000 000 000 01			Hydrog.atom
trillionth	10^{-12} 0.000 000 000 001	picometer	p	
	10^{-13} 0.000 000 000 000 1			
	10^{-14} 0.000 000 000 000 01			nucleus
quadrilliont	10^{-15} 0.000 000 000 000 001	femtometer	f	
	10^{-16} 0.000 000 000 000 000 1			quark

INVERSE OF A 2X2 MATRIX:

Reverse the positions of a and d, and change the signs of c and b. This is called the **adjoint** or adj of the matrix.

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad adj\ M = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \qquad \det(M) = Z = ad - cb$$

$$M^{-1} = \frac{1}{Z} \cdot adj\ M \qquad or \qquad M^{-1} = \frac{1}{ad - cb} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

INVERSE OF A 3X3 MATRIX:

$$N = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix}$$

To find the determinant Z: copy the first 2 columns and multiply the cells diagonally,

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix} \begin{matrix} a & b \\ d & e \\ h & k \end{matrix}$$

$$Z = (aek + bfh + cdk) - (ceg + afh + bdk)$$

Transpose the matrix (The rows become the columns). Then change the signs of the second column and the second row end cells.

$$N = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix} \qquad N^T = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & k \end{bmatrix} \quad chnage\ signs\ as\ shown: \begin{bmatrix} a & -d & g \\ -b & e & -h \\ c & -f & k \end{bmatrix}$$

Calculate the cells of the inverse matrix. Each cell is the determinant of the 2X2 matrix formed by the cells that are not in its row or column.

$$A = [(ek - (-f)(-h)] \qquad D = [(-b)k - c(-h)] \qquad G = [(-b)(-f) - ce]$$

$$B = [(-d)k - (-f)g] \qquad E = [ak - cg] \qquad H = [a(-f) - c(-d)]$$

$$C = [(-d)h - eg] \qquad F = [a(-h) - (-b)g] \qquad K = [ae - (-b)(-d)]$$

The inverse matrix is obtained by dividing the adjoint by the determinant.

$$adj\ N = \begin{bmatrix} A & D & G \\ B & E & H \\ C & F & K \end{bmatrix} \qquad N^{-1} = \frac{1}{Z} \begin{bmatrix} A & D & G \\ B & E & H \\ C & F & K \end{bmatrix}$$

INVERSE OF 3X3 MATRIX NUMERICAL EXAMPLE:

$$N = \begin{bmatrix} -3 & 5 & 1 \\ 1 & 1 & -2 \\ 2 & -1 & 6 \end{bmatrix}$$

Copy the 2 first columns and multiply the cells diagonally to obtain Z,

$$\begin{bmatrix} -3 & 5 & 1 \\ 1 & 1 & -2 \\ 2 & -1 & 6 \end{bmatrix} \begin{matrix} -3 & 5 \\ 1 & 1 \\ 2 & -1 \end{matrix} \qquad \det(N) = Z$$

$$Z = [(-3)\cdot 1\cdot 6 + 5(-2)\cdot 2 + 1\cdot 1(-1)] - [1\cdot 1\cdot 2 + (-3)(-2)(-1) + 5\cdot 1\cdot 6] = -65$$

$$N = \begin{bmatrix} -3 & 5 & 1 \\ 1 & 1 & -2 \\ 2 & -1 & 6 \end{bmatrix} \qquad N^T = \begin{bmatrix} -3 & 1 & 2 \\ 5 & 1 & -1 \\ 1 & -2 & 6 \end{bmatrix} \qquad change\ signs: \begin{bmatrix} -3 & -1 & 2 \\ -5 & 1 & 1 \\ 1 & 2 & 6 \end{bmatrix}$$

$$A = [(1\cdot 6 - 2\cdot 1] = 4 \qquad D = [(-5)6 - 1\cdot 1] = -31 \qquad G = [(-5)2 - 1\cdot 1] = -11$$

$$B = [(-1)6 - 2\cdot 2] = -10 \qquad E = [(-3)6 - 1\cdot 2] = -20 \qquad H = [(-3)2 - (-1)1] = -5$$

$$C = [(-1)1 - 1\cdot 2] = -3 \qquad F = [(-3)1 - (-5)2] = 7 \qquad K = [(-3)1 - (-5)(-1)] = -8$$

$$N^{-1} = \frac{1}{Z}\begin{bmatrix} A & D & G \\ B & E & H \\ C & F & K \end{bmatrix} \qquad adj\ N = \begin{bmatrix} A & D & G \\ B & E & H \\ C & F & K \end{bmatrix} = \begin{bmatrix} 4 & -31 & -11 \\ -10 & -20 & -5 \\ -3 & 7 & -8 \end{bmatrix}$$

$$N^{-1} = \frac{-1}{65}\begin{bmatrix} 4 & -31 & -11 \\ -10 & -20 & -5 \\ -3 & 7 & -8 \end{bmatrix} = \begin{bmatrix} \frac{-4}{65} & \frac{31}{65} & \frac{11}{65} \\ \frac{2}{13} & \frac{4}{13} & \frac{1}{13} \\ \frac{3}{65} & \frac{-7}{65} & \frac{8}{65} \end{bmatrix}$$