GEOMETRY
REFERENCE CHART

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POINTS, LINES & PLANES

A **POINT** is an exact location in space with no size.

Points A *or* B

A **LINE** is an infinite set of points forming a straight path extending in two opposite directions.

$\overleftrightarrow{PQ}$ *or Line* k

Two points determine one and only one line. Points are **COLLINEAR** if they lie on one line.

A **SEGMENT** is the part of a line between two points.

Segment $\overline{MN}$

A **RAY** is the part of a line that starts at one point and extends indefinitely in the other direction .

Ray $\overrightarrow{CD}$

A **PLANE** is a flat surface extending indefinitely in two directions.

Plane ***RST*** *or* f

Three points determine one and only one plane. Points (***R, S*** and ***T***) are **COPLANAR** if they lie on one plane.

An **INTERSECTION** is the point or set of points common to two figures. Two lines intersect at a point. Two Planes intersect at a line.

$\overleftrightarrow{k}$ *and* $\overleftrightarrow{h}$ *intersect at* Q

M *and* N *intersect at* $\overleftrightarrow{AB}$

The **MIDPOINT** (***E***) divides a segment into two congruent segments.

CONGRUENT parts have equal measurements.

$\overline{DE} \cong \overline{EF}$

$m\overline{DE} = m\overline{EF}$

PARALLEL LINES are coplanar lines that do not intersect.

SKEW LINES are lines that do not intersect and are not coplanar.

PARALLEL PLANES extend indefinitely and do not intersect.

Parallel planes $\overleftrightarrow{N} \parallel \overleftrightarrow{M}$

$\overleftrightarrow{h}$ *and* $\overleftrightarrow{PQ}$ *are skew lines*

h and k are parallel $\overleftrightarrow{h} \parallel \overleftrightarrow{k}$

Line ***g*** is **PERPENDICULAR** to line ***h***, therefore all 8 angles are right angles.

TRANSVERSAL LINE ***f*** intersects parallel lines ***h*** and ***k*** forming 8 angles.

$\hat{a} = \hat{c} = \hat{m} = \hat{p}$; $\hat{b} = \hat{d} = \hat{n} = \hat{q}$

$\hat{1} = \hat{2} = \hat{3} = \hat{4} = \hat{5} = \hat{6} = \hat{7} = \hat{8} = 90°$

ALTERNATE INTERIOR ANGLES
(***d*** and ***n***, ***c*** and ***m***) *are congruent.*

ALTERNATE EXTERIOR ANGLES
(***a*** and ***p***, ***b*** and ***q***) *are congruent.*

CORRESPONDING ANGLES
(***a*** and ***m***, ***b*** and ***n***, ***d*** and ***q***, ***c*** and ***p***) *are congruent.*

CONSECUTIVE INTERIOR ANGLES
(***d*** and ***m***, ***c*** and ***n***) *are supplementary.*

ANGLES

An **ANGLE** (α) is formed by two **SIDES** (***SR*** and ***ST***) joined at a **VERTEX** (***S***).

An **ACUTE ANGLE** is less than 90° $m\angle\alpha < 90°$

A **RIGHT ANGLE** is 90° $m\angle\beta = 90°$

An **OBTUSE ANGLE** is more than 90° $m\angle\gamma > 90°$

A **STRAIGHT ANGLE** is 180° $m\angle\delta = 180°$

COMPLEMENTARY ANGLES add to 90°

$m\angle ROP + m\angle QOR = 90°$

Angle ***QOR*** is **ADJACENT** to angle ***ROP.***

SUPPLEMENTARY ANGLES add to 180°

$m\angle KLH + m\angle KLM = 180°$

VERTICAL ANGLES are congruent (***WVZ*** and ***XVY***).

$m\angle WVZ = m\angle XVY$

Ray ***EO*** is the **BISECTOR** of angle ***GOH***. Angles ***GOE*** and ***EOH*** are congruent.

$\overline{MN} \cong \overline{NL}$ $\quad \angle GOE \cong \angle EOH$

KH is **PERPENDICULAR BISECTOR** of segment ***ML***. Segments ***MN*** and ***NL*** are congruent.

TRIANGLES

A triangle is **EQUILATERAL** if its 3 sides are congruent .

A triangle is **EQUIANGULAR** if its 3 angles are congruent.

A triangle is **ACUTE** if its 3 angles are acute

Triangle ***GHK*** is equilateral, equiangular and acute.

A triangle is **SCALENE** if none of its sides are congruent.

A triangle is **OBTUSE** if one of its angles is obtuse.

Triangle ***OPQ*** is scalene and obtuse.

A triangle is **RIGHT** if it has one right angle (triangle ***LMN***).

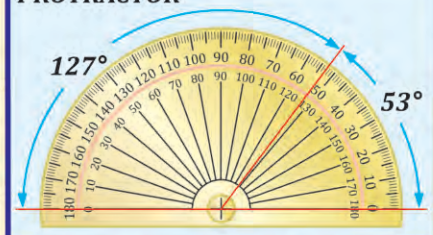
A triangle is **ISOSCELES** if it has two congruent sides and two congruent base angles (triangle ***DEF***).

The 3 **INTERIOR ANGLES** of a triangle add to 180° (α,β,γ). The sum of the **REMOTE INTERIOR ANGLES** (α,β) equal the **EXTERIOR ANGLE** (δ).

$\angle\alpha + \angle\beta + \angle\gamma = 180°$

$\angle\alpha + \angle\beta = \angle\delta$

PROTRACTOR



FLAT FIGURES

A=area; P=perimeter

SQUARE
$A = a^2$
$P = 4a$

RECTANGLE
$A = bh$
$P = 2(b + h)$

TRAPEZOID
$A = h\left(\frac{a+b}{2}\right)$
$P = a + b + c + d$

PARALLELOGRAM
$A = bh$
$P = 2(b + c)$

RHOMBUS
$A = \frac{1}{2}mn$
$P = 4c$

TRIANGLE
$A = \frac{1}{2}bh$
$P = a + b + c$

PENTAGON
$A = \frac{5}{2}as$
$P = 5s$

CIRCLE
$A = \pi r^2$
$C = 2\pi r$

SECTOR $A = \frac{\theta r^2}{2}$

ARC $s = \theta r$

SOLIDS

$A_b = base\ area$

$S = surface\ area$

$V = volume$

CUBE
$S = 6a^2$
$V = a^3$

RECTANGULAR PRISM
$A_b = zw$

SLANTED PRISM

PENTAGONAL PRISM
$A_b = \frac{5}{2}az$

CYLINDER
$A_b = \pi r^2$
$S = 2(A_b + \pi rh)$

$V = A_b h$

TRIANGULAR PRISM
$A_b = zw$
$V = \frac{1}{2}A_b h$

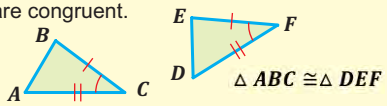
SQUARE PYRAMID
$A_b = a^2$
$S = a^2 + 2ka$
$V = A_b h/3$

SPHERE
$S = 4\pi r^2$
$V = \frac{4}{3}\pi r^3$

CONE
$S = A_b + \pi rk$
$A_b = \pi r^2$
$V = A_b h/3$

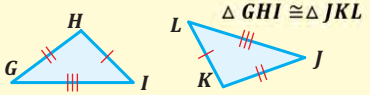
CONGRUENT TRIANGLES

SAS: two sides and their included angle are congruent.



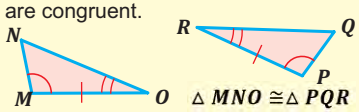
$\triangle ABC \cong \triangle DEF$

SSS: three sides are congruent.



$\triangle GHI \cong \triangle JKL$

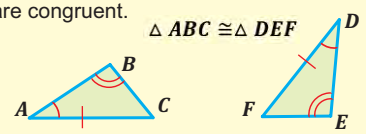
ASA: two angles and their included side are congruent.



$\triangle MNO \cong \triangle PQR$

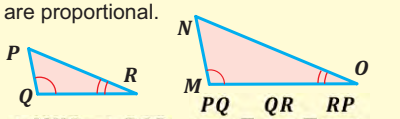
AAS: two angles and a non included side are congruent.

$\triangle ABC \cong \triangle DEF$



PROPORTION & SIMILARITY

Congruent triangles are also similar. Triangles with two congruent angles are **SIMILAR TRIANGLES**, and their sides are proportional.



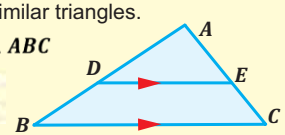
$\triangle MNO \sim \triangle PQR$

$$\frac{PQ}{NM} = \frac{QR}{MO} = \frac{RP}{ON}$$

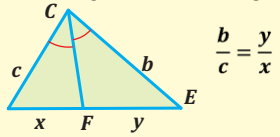
A line parallel to one side divides the other sides into proportional parts and forms two similar triangles.

$\triangle ADE \sim \triangle ABC$

$$\frac{AD}{DB} = \frac{AE}{EC}$$



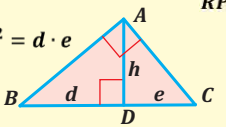
CF is the angle bisector of angle ***DCE***.



$$\frac{b}{c} = \frac{y}{x}$$

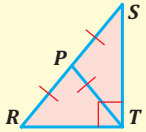
h is the altitude of right triangle ***ABC***.
P is the hypotenuse mid point of right triangle ***RST***.

$h^2 = d \cdot e$



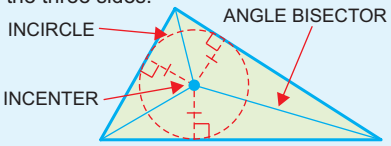
$\triangle ABC \sim \triangle DAC \sim \triangle DAB$

$RP = PS = PT$

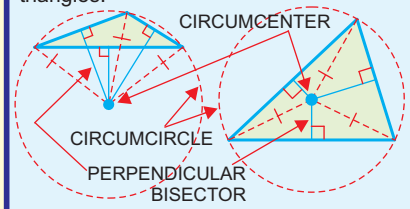


TRIANGLE CENTERS

The **INCENTER** is the intersection point of the three angle bisectors and the **INCIRCLE** center. It is equidistant from the three sides.



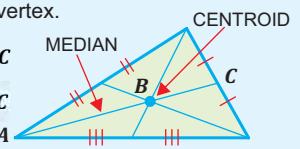
The **CIRCUMCENTER** is the intersection point of the three perpendicular bisectors and the center of the **CIRCUMCIRCLE**. It is equidistant from the three vertexes. The circumcenter is outside of obtuse triangles.



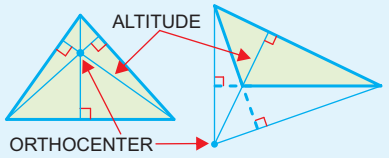
The **CENTROID** is the intersection point of the three medians. The **MEDIAN** is a line joining a side midpoint with the opposite vertex.

$AB = \frac{2}{3} AC$

$BC = \frac{1}{3} AC$

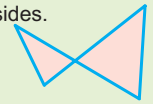


The **ORTHOCENTER** is the intersection point of the three altitudes. The orthocenter is outside of obtuse triangles.



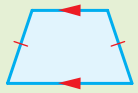
QUADRILATERALS

A **QUADRILATERAL** is a figure with four sides.

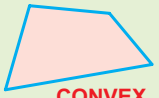


COMPLEX: with intersecting sides.

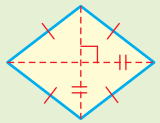
CONCAVE: one interior angle > 180°



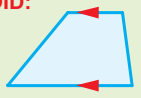
ISOSCELES TRAPEZOID: has two congruent legs.



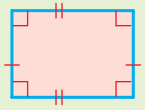
CONVEX



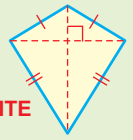
RHOMBUS



TRAPEZOID

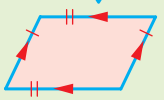


RECTANGLE

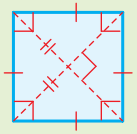


KITE

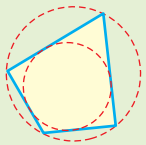
INSCRIPTIBLE: with an inscribed circle.



PARALLELOGRAM

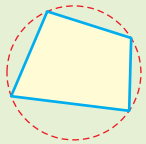


SQUARE



BICENTRIC: with circumscribed and inscribed circles.

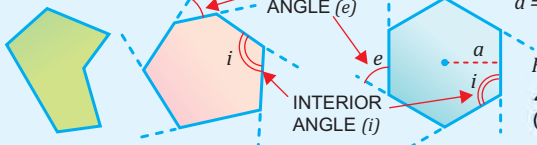
CYCLIC: with a circumscribed circle.



PLATONIC SOLIDS

Only 5 solids can be constructed using regular equilateral polygons.

POLYGONS



n = number of sides
a = **APOTHEM:** *line from center to side mid point.*

For all concave polygons:

$\angle e \cdot n = 360°$

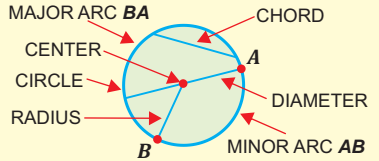
$(n-2) \cdot 180° = \sum \angle i$

CONCAVE **CONVEX** **IRREGULAR** **REGULAR:** congruent sides and angles.

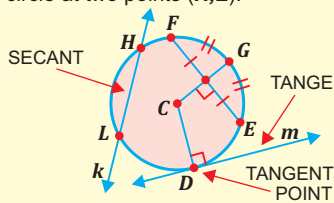
PENTAGON. **HEXAGON** **HEPTAGON** **OCTAGON** **NONAGON** **DECAGON**

CIRCLES

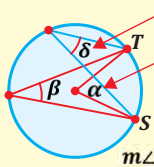
A **CIRCLE** is a set of coplanar points equidistant from a **CENTER** point. The **RADIUS** is a segment connecting the center and a point on the circle. A **CHORD** is a segment connecting two circle points. The **DIAMETER** is a chord containing the center. An **ARC** is a part of the circle between two points.



A **TANGENT** is a line (***m***) intersecting a circle at one point only (***D***).
A **SECANT** is a line (*k*) intersecting a circle at two points (***H,L***).



The radius ***CG*** is perpendicular to cord ***EF***, bisects it and its arc ***EGF***.
A radius to the **TANGENTIAL POINT** (***D***) is perpendicular to the tangent (***m***).



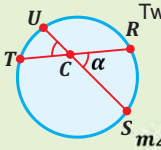
$m\angle\alpha = m\widehat{TS}$

$m\angle\beta = m\angle\delta$

$m\angle\alpha = 2m\angle\beta = 2m\angle\delta$

Two radii form a **CENTRAL ANGLE** (α). Two chords with a common circle point form an **INSCRIBED ANGLE** (β, δ). The arc angle (***TS***) is equal to its central angle (α).

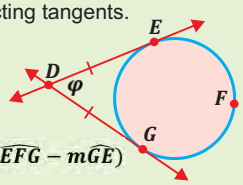
Two chords intersecting inside a circle.



$UC \cdot CS = TC \cdot CR$

$m\angle\alpha = \frac{1}{2}(m\widehat{UT} + m\widehat{RS})$

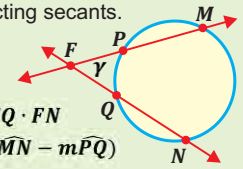
Two intersecting tangents.



$DE = DG$

$m\angle\varphi = \frac{1}{2}(m\widehat{EFG} - m\widehat{GE})$

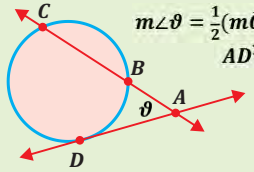
Two intersecting secants.



$FP \cdot FM = FQ \cdot FN$

$m\angle\gamma = \frac{1}{2}(m\widehat{MN} - m\widehat{PQ})$

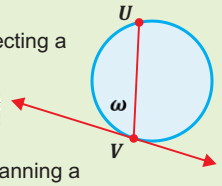
An intersecting secant and tangent.



$m\angle\vartheta = \frac{1}{2}(m\widehat{CD} - m\widehat{DB})$

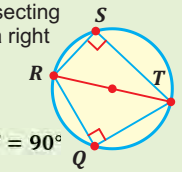
$AD^2 = AB \cdot AC$

A chord intersecting a tangent.



$m\angle\omega = \frac{1}{2}m\widehat{UV}$

Two chords spanning a diameter and intersecting at the circle, form a right angle.



$m\angle RST = m\angle RQT = 90°$