SELECTED CALCULUS EQUATIONS

Partial fractions

$Q(x) = (x-a)(x-b)(x-c)\ldots \quad \frac{P(x)}{Q(x)} = \frac{A_1}{(x-a)} + \frac{A_2}{(x-b)} + \frac{A_3}{(x-c)} + \cdots$

$Q(x) = (x-a)^n \quad \frac{P(x)}{Q(x)} = \frac{A_1}{(x-a)} + \frac{A_2}{(x-a)^2} + \frac{A_3}{(x-a)^3} + \cdots \frac{A_n}{(x-a)^n}$

$Q(x) = ax^2 + bx + c \quad \frac{P(x)}{Q(x)} = \frac{Ax+B}{ax^2+bx+c}$

$Q(x) = (ax^2+bx+c)^n \quad \frac{P(x)}{Q(x)} = \frac{A_1x+B_1}{ax^2+bx+c} + \frac{A_2x+B_2}{(ax^2+bx+c)^2} + \frac{A_3x+B_3}{(ax^2+bx+c)^3} + \cdots \frac{A_nx+B_n}{(ax^2+bx+c)^n}$

Linearization

To approximate $f(x)$ *near* $f(a)$: $L(a) = f(a) + f'(a)(x-a)$

Integrating by parts

$$\int u\,dv = uv - \int v\,du$$

For: $x^a e^{bx}$, $x^a \sin bx$, $x^a \cos bx$, $\rightarrow x^a = u$ *the rest* $= dv$

For: $x^a \ln(bx)$, $x^a \sin^{-1} bx$, $x^a \cos^{-1} bx$, $x^a \tan^{-1} bx$, $x^a \cot^{-1} bx \rightarrow x^a dx = dv$ *the rest* $= u$

For: $e^{ax} \sin bx$, $e^{ax} \cos bx \rightarrow e^{ax} = u$ *or* $e^{ax} dx = dv$

Trigonometric substitutions

$x = a\sin\vartheta \quad \int \sqrt{a^2-x^2}\,dx = \int a^2\cos^2\vartheta\,d\vartheta = a^2\left(\frac{1}{2}\vartheta + \frac{1}{4}\sin 2\vartheta\right) + C$

$x = a\tan\vartheta \quad \int \sqrt{a^2+x^2}\,dx = \int a^2\sec^3\vartheta\,d\vartheta = a^2\left(-\frac{1}{2}\sec\vartheta\tan\vartheta + \frac{1}{2}\ln|\sec\vartheta + \tan\vartheta|\right) + C$

$x = a\sec\vartheta \quad \int \frac{\sqrt{x^2-a^2}}{x}\,dx = \int a\tan^2\vartheta\,d\vartheta = a(\tan\vartheta - \vartheta) + C$

Center of gravity

$\bar{x} = \frac{1}{A}\int_a^b xf(x)dx \quad \bar{y} = \frac{1}{A}\int_a^b \frac{1}{2}[f(x)]^2 dx \quad A = \int_a^b f(x)dx$

Power series

$$f(x)_{at\,x_o} = \sum_0^\infty \frac{f^n(x_0)}{n!}(x-x_0)^n = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^3 + \cdots + \frac{f^n(x_0)}{n!}(x-x_0)^n + \cdots$$

$$f(x)_{at\,x_o=0} = \sum_0^\infty \frac{f^n(0)}{n!}x^n = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots + \frac{f^n(0)}{n!}x^n + \cdots$$

Binomial theorem:

$$(x+y)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k}y^k = \binom{n}{0}x^n y^0 + \binom{n}{1}x^{n-1}y^1 + \binom{n}{2}x^{n-2}y^2 + \cdots \binom{n}{n-1}x^1 y^{n-1} + \binom{n}{n}x^0 y^n \qquad \binom{n}{k} = \frac{n!}{(n-k)!\cdot k!}$$

$$(1+x)^n = \sum_{k=0}^n \binom{n}{k} x^k = \binom{n}{0}x^0 + \binom{n}{1}x^1 + \binom{n}{2}x^2 + \cdots \binom{n}{n-1}x^{n-1} + \binom{n}{n}x^n = 1 + \sum_{k=1}^\infty \frac{n(n-1)\cdots(n-k+1)}{k!}x^k \quad |x| < 1$$

Common Maclaurin series:

$$\frac{1}{1-x}=\sum_{0}^{\infty}x^n=1+x+x^2+x^3+\cdots \qquad R=1$$

$$\frac{1}{1-x^2}=\sum_{0}^{\infty}(-1)^n x^{2n}=1-x^2+x^4-x^6+\cdots \qquad R=1$$

$$e^x=\sum_{0}^{\infty}\frac{x^n}{n!}=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\cdots \qquad R=\infty$$

$$\sin x=\sum_{0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots \qquad R=\infty$$

$$\cos x=\sum_{0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots \qquad R=\infty$$

$$\tan^{-1}x=\sum_{0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)}=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\cdots \qquad R=1$$

$$\ln(1+x)=\sum_{1}^{\infty}(-1)^{n+1}\frac{x^n}{n!}=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots \qquad R=1$$

Remainder:

$$|R_n(x)|\le\frac{M}{(n+1)!}|x-x_0|^{n+1}$$

Geometric series

$$\sum_{n=0}^{\infty}a\cdot r^n=\frac{a}{1-r}$$

Alternating harmonic series

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots=ln2$$

Binomial series

$$(1+x)^m=1+\sum_{k=1}^{\infty}\frac{m(m-1)\cdots(m-k+1)}{k!}x^k \qquad |x|<1$$

Convergence

$$If\ \lim_{n\to\infty}a_n=0\Rightarrow\sum_{1}^{\infty}a_n\ converges$$

$$\sum_{1}^{\infty}a_n\ converges\ \Leftrightarrow\ \int_1^{\infty}f(x)dx\ converges$$

Fundamental theorem of calculus

$$g(x)=\int_a^{u(x)}f(t)dt \quad g'(x)=f(u)u'$$

Limits

$$\lim_{x\to0}\frac{\sin x}{x}=1 \qquad \lim_{x\to0}\frac{1-\cos x}{x}=0$$

$$\lim_{x\to\infty}\frac{\sin x}{x}=0 \qquad \lim_{x\to\infty}\frac{\cos x}{x}=0$$

Areas and length of curves

Rectangular:

$$A=\int_a^b f(x)dx \qquad L=\int_a^b\sqrt{1+f'(x)^2}\,dx$$

Parametric:

$$A=\int_{t_1}^{t_2}g(t)\cdot f(t)dt \qquad L=\int_{t_1}^{t_2}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$$

Polar:

$$A=\int_\alpha^\beta\frac{1}{2}r^2d\vartheta \qquad r=f(\vartheta) \qquad L=\int_\alpha^\beta\sqrt{r^2+\left(\frac{dr}{d\vartheta}\right)^2}\,d\vartheta$$

Special Products

$$(a+b)^2=a^2+2ab+b^2$$

$$(a-b)^2=a^2-2ab+b^2$$

$$a^2-b^2=(a+b)(a-b)$$

$$a^2+b^2=(a+bi)(a-bi)$$

$$a^3+b^3=(a+b)(a^2-ab+b^2)$$

$$a^3-b^3=(a-b)(a^2+ab+b^2)$$

$$a^4+b^4=\left(a^2+\sqrt{2}ab+b^2\right)\left(a^2-\sqrt{2}ab+b^2\right)$$

$$a^4-b^4=\left(a^2+b^2\right)(a+b)(a-b)$$

Exponential

Newton law of cooling: $T(t)=T_s+(T_i-T_s)e^{-kt}$

Logistic growth: $y=\dfrac{M}{1+ce^{-kMt}}$ *upper limit*: M

Half life: $A=A_0\left(\frac{1}{2}\right)^{\frac{t}{HL}}$ $t=HL\cdot\log_{\frac{1}{2}}\left(\frac{A}{A_0}\right)$

Newton's method

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$$

Summation

$$\sum_{k=1}^{n}k=\frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n}k^2=\frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n}k^3=\left[\frac{n(n+1)}{2}\right]^2$$

$$\sum_{k=1}^{n}ar^{k-1}=a\frac{1-r^n}{1-r} \qquad r\ne0,1 \quad geometric\ series$$

$$\sum_{k=1}^{\infty}ar^{k-1}=\frac{a}{1-r} \qquad |r|<1 \quad infinite\ geometric\ series$$

Vectors

$\boldsymbol{u} \circ \boldsymbol{v} = \boldsymbol{u}_x \boldsymbol{v}_x + \boldsymbol{u}_y \boldsymbol{v}_y + \boldsymbol{u}_z \boldsymbol{v}_z$ *dot product*

$\boldsymbol{u} \circ \boldsymbol{v} = \|\boldsymbol{u}\|\|\boldsymbol{v}\| \cos \boldsymbol{\vartheta} \qquad \cos \boldsymbol{\vartheta} = \dfrac{\boldsymbol{u} \circ \boldsymbol{v}}{\|\boldsymbol{u}\|\|\boldsymbol{v}\|}$

$\boldsymbol{proj_b v} = \dfrac{\boldsymbol{v} \circ \boldsymbol{b}}{\|\boldsymbol{b}\|^2} \cdot \boldsymbol{b} \qquad \boldsymbol{scalar\ proj} = \dfrac{\boldsymbol{v} \circ \boldsymbol{b}}{\|\boldsymbol{b}\|}$

$\boldsymbol{W} = \|\boldsymbol{F}\|\,\|\boldsymbol{D}\| \cos \boldsymbol{\vartheta} = \boldsymbol{F} \circ \boldsymbol{D} \quad \boldsymbol{W}: work \;\; \boldsymbol{F}: force\ \boldsymbol{D}: displcmt$